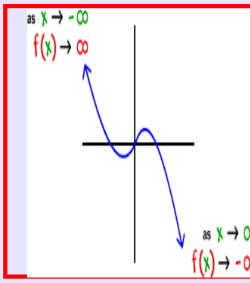
Math 245

Spring 2022

Lecture 21



Class QZ 8

Given $\boxed{f(x) = 3x - 8}$, find its inverse.

$y = 3x - 8$

$x = 3y - 8$

$x + 8 = 3y$ $\quad y = \dfrac{x+8}{3}$

$$\boxed{f^{-1}(x) = \frac{x+8}{3}}$$

$f^{(-1)}$ Not the inverse

" f inverse of x "

$f(3) = 3(3) - 8 = 1$ $\quad f^{-1}(1) = \dfrac{1+8}{3} = 3$

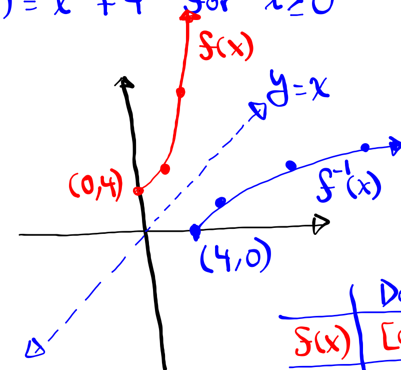
$f(0) = 3(0) - 8 = -8$ $\quad f^{-1}(-8) = \dfrac{-8+8}{3} = 0$

$f(2) = 3(2) - 8 = -2$ $\quad f^{-1}(-2) = \dfrac{-2+8}{3} = 2$

Given $f(x) = x^2 + 4$ for $x \geq 0$

x	y
0	4
1	5
2	8
3	13

...



Function by V.L.T.

It has an inverse by H.L.T.

	Domain	Range
$f(x)$	$[0,\infty)$	$[4,\infty)$
$f^{-1}(x)$	$[4,\infty)$	$[0,\infty)$

Find $f^{-1}(x)$

$f(x) = x^2 + 4$

$y = x^2 + 4$

$x = y^2 + 4$

$x - 4 = y^2$

square root of both Sides

$\sqrt{x-4} = \sqrt{y^2}$ $y \geq 0$

$\sqrt{x-4} = y$ $\boxed{f^{-1}(x) = \sqrt{x-4}}$

Find $f^{-1}(x)$ for $f(x) = x^3 - 4$.

$f(x) = x^3 - 4$

$y = x^3 - 4$

$x = y^3 - 4$

$x + 4 = y^3$

Cube root of both Sides

$\sqrt[3]{x+4} = \sqrt[3]{y^3}$

$\sqrt[3]{x+4} = y \Rightarrow \boxed{f^{-1}(x) = \sqrt[3]{x+4}}$

March 16, 2022

Given $f(x) = \frac{4}{x-2}$

$x - 2 \neq 0$

$x \neq 2$

Domain: All reals except 2.

Find $f^{-1}(x)$

$f(x) = \frac{4}{x-2}$ $\quad y = \frac{4}{x-2}$ $\quad x = \frac{4}{y-2}$

Cross-Multiply $\quad x(y-2) = 4$

$xy - 2x = 4$

$xy = 2x + 4$

$y = \frac{2x+4}{x}$

$$\boxed{f^{-1}(x) = \frac{2x+4}{x}}$$

Domain: All reals except 0

	Domain	Range
$f(x)$	$(-\infty,2) \cup (2,\infty)$	$(-\infty,0) \cup (0,\infty)$
$f^{-1}(x)$	$(-\infty,0) \cup (0,\infty)$	$(-\infty,2) \cup (2,\infty)$

Consider $f(x) = \frac{x}{x+4}$

$x + 4 \neq 0$

$x \neq -4$

Domain: All reals except $-4 \Rightarrow (-\infty,-4) \cup (-4,\infty)$

Find $f^{-1}(x)$

$f(x) = \frac{x}{x+4}$ $\quad y = \frac{x}{x+4}$ $\quad x = \frac{y}{y+4}$

Cross-Multiply

$\frac{x}{1} = \frac{y}{y+4}$

$\boxed{x(y+4)} = \boxed{1 \cdot y}$

$xy + 4x = y$

$xy - y = -4x$

$y(x-1) = -4x$

$y = \frac{-4x}{x-1}$

$$\boxed{f^{-1}(x) = \frac{-4x}{x-1}}$$

	Domain	Range
$f(x)$	$(-\infty,4) \cup (4,\infty)$	$(-\infty,1) \cup (1,\infty)$
$f^{-1}(x)$	$(-\infty,1) \cup (1,\infty)$	$(-\infty,4) \cup (4,\infty)$

Domain: All reals except 1

$x - 1 \neq 0$

$x \neq 1$

$(-\infty,1) \cup (1,\infty)$

Review of radicals

$\sqrt[\text{Index}]{\text{Radicand}}$

Index (No index $\Rightarrow$ 2)

Radicand

$\sqrt{3x-5}$

Index = 2

Radicand = $3x-5$

$\sqrt[3]{x^2-2x+5}$

Index = 3

Radicand = x^2-2x+5

when index is even $\Rightarrow$ Radicand ≥ 0

when index is odd $\Rightarrow$ No restrictions on the radicand.

$f(x) = \sqrt{x-4}$

No index $\Rightarrow$ index = 2 $\Rightarrow$ even index

Radicand ≥ 0 $\quad x-4 \geq 0 \quad x \geq 4$

4

Domain $\Rightarrow$ $[4, \infty)$

$f(x) = \sqrt[3]{x+2}$

Index = 3

Radicand = $x+2$

Index is odd $\Rightarrow$ No restrictions on the radicand

Domain $(-\infty, \infty)$

$f(x) = \sqrt{x^2 - 25}$

Index = No index ⇒ index = 2 ← even index

Radicand: $x^2 - 25$

Find domain $x^2 - 25 \geq 0$

$(x+5)(x-5) \geq 0$

$x+5=0$ $x-5=0$

$x=-5$ $x=5$

x	$-\infty$	-5	5	∞
$x+5$	$-$	$+$	$+$	
$x-5$	$-$	$-$	$+$	
Radicand	$+$	$-$	$+$	

$\boxed{(-\infty, -5] \cup [5, \infty)}$ → Domain for $f(x)$.